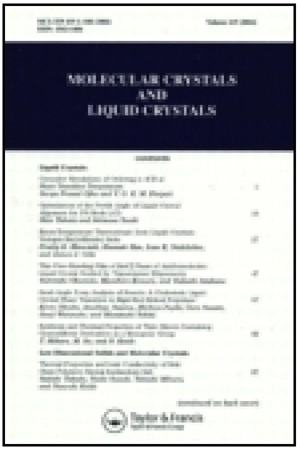

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Lifshitz Point in Phase Diagrams of Cholesteric Films in Electric Fields

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LIFSHITZ POINT IN PHASE DIAGRAMS OF CHOLESTERIC FILMS IN ELECTRIC FIELDS

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Abstract A mathematical procedure for locating Lifshitz points in phase diagrams of field induced transitions in cholesteric films is presented.

INTRODUCTION

As well known, planar cholesteric films can exhibit either a Freedericksz transition or a transition to a stripe texture[1] in electric fields. Periodic equilibrium structures are also found in non-twisted nematic layers with large elastic anisotropy[2].

Thus three phases have to be taken into consideration, namely the initial director configuration (I), a homogeneously distorted state (II) due to a Freedericksz transition and a modulated phase (III). Recently, Allender claimed[3], that these three phases meet at a Lifshitz point of a suitably constructed phase diagram. In this communication a mathematical procedure for obtaining the position of Lifshitz point is presented.

Figure 1 illustrates the geometry of a planar cholesteric film. The director is fixed at the plate surfaces ($x = 0$ and $x = d$) and twisted by an azimuthal angle α. Below the threshold voltage of an instability the angle Θ_m enclosed by the director and the mid-plane of the film $x = d/2$ is zero.

The stripes are perpendicular to the wave vector q of the periodic distortions. q and the director at the lower plate enclose a definite angle $\varkappa$.

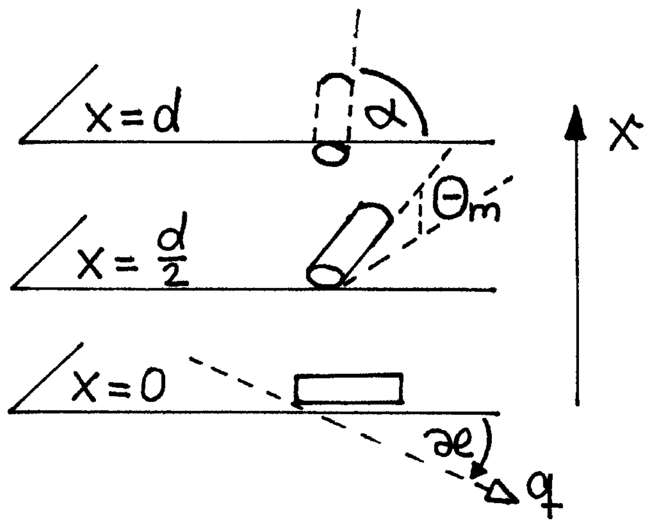

FIGURE 1 Geometry of a cholesteric layer

For convenience the abbreviations

$$k_2 = \frac{K_{22}}{K_{11}}, \quad k_3 = \frac{K_{33}}{K_{11}}, \quad \omega = \frac{\alpha}{\pi}$$

$$\beta = \frac{2\pi d}{P\alpha} \quad \text{and} \quad \gamma = \frac{\varepsilon_{\parallel} - \varepsilon_{\perp}}{\varepsilon_{\perp}}$$

are introduced. K_{11}, K_{22} and K_{33} are elastic constants defined in the frame-work of the Oseen-Frank theory[4].

$\varepsilon_{\parallel}$ and $\varepsilon_{\perp}$ are dielectric constants measured parallel and perpendicular to the director, respectively.

PHASE DIAGRAM WITH LIFSHITZ POINT

Let us choose the applied voltage U and k_2 as independent variables. If a Lifshitz point L exists, a phase diagram shown in Figure 2 can be constructed.

Three phases characterized by

(I) $\Theta_m = 0$, (II) $\Theta_m = \text{constant} \neq 0$

and a modulated one

(III) $\Theta_m \sim \cos(qy)$

meet at the point L. (The y-axis is parallel to q in Figure 1.)

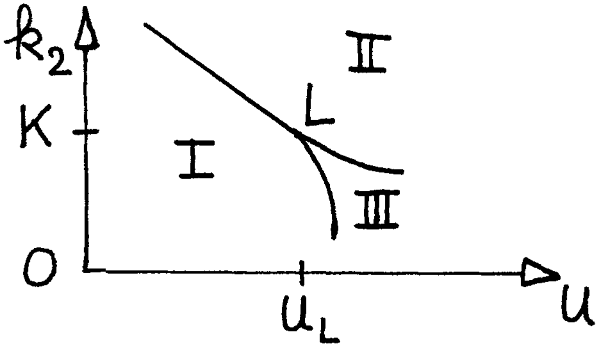

FIGURE 2 Phase diagram with Lifshitz point L

LOCATION OF THE LIFSHITZ POINT

The critical voltage at the Lifshitz point is

$$U_L = \sqrt{\frac{\pi^2 K_{11}}{\varepsilon_{\parallel} - \varepsilon_{\perp}} \left[1 + \omega^2 (k_3 - 2K + 2K\beta)\right]}$$

and the critical value K of k_2 is obtained by a simple procedure.

Introducing an integral operator IN by the identity

$$\ln(f(x)) = -\frac{\pi - x}{\pi}\int_0^x \xi f(\xi)d\xi - \frac{x}{\pi}\int_x^{\pi}(\pi-\xi)f(\xi)d\xi$$

we define

$$F(x) = \ln(f(x)) \text{ and } G(x) = \ln(g(x))$$

where

$$f(x) = \frac{1-K}{K}\sin\Omega\cos x - \frac{k_3-K+2K\beta}{K}\omega\cos\Omega\sin x\,,$$

$$g(x) = \gamma\cos\Omega\sin x \qquad \text{and}$$

$$\Omega = \omega x + \varkappa .$$

Now a function D ($\varkappa$) is obtained by integration

$$D(\varkappa) = \frac{1}{\pi}\int_0^{\pi} dx\,\sin x\,\Big\{-(1-K)\sin\Omega\,\frac{dF}{dx}$$

$$-(1-2K+k_3+2K\beta)\,\omega\cos\Omega\,F$$

$$+(K\sin^2\Omega + k_3\cos^2\Omega)\sin x$$

$$-\left[1+\omega^2(k_3-2K+2K\beta)\right]\cos\Omega\,G\Big\}.$$

Let D($\varkappa$) have its absolute minimum for $\varkappa = \varkappa_m$.
If $\varkappa_m$ obeys

$$\varkappa_m = \begin{cases} 0 \text{ or } \frac{\pi}{2} & \text{if } \alpha = 0 \\ -\frac{\alpha}{2} + n\pi & \text{if } \alpha \neq 0 \end{cases}$$

(n is an integer number),

then a Lifshitz point exists and K is obtained by the equation

$$D(\varkappa_m;\,K) = 0 .$$

For example, if $\alpha = \beta = 0$, we get $\varkappa_m = \frac{\pi}{2}$ and $K \simeq 0.303$ in accordance with a result of Lonberg and Meyer[2].

Secondly, let us regard a twisted cholesteric film with $\alpha = 180^{o}$. In this case also results $\mathcal{H}_m = \pi/2$, and K satisfies the equation

$$4K^2 + 12Kk_3 + 8(1-K)^2 - 3(k_3-1+2K\beta)^2$$
$$-2\left(\frac{\pi^2+3}{3}\right)(k_3+1-2K+2K\beta)^2$$
$$+\left(\frac{2\pi^2+15}{3}\right)(k_3+1-2K+2K\beta)K\gamma = 0.$$

It should be remarked, that a previously obtained formula[5] for K refers to a small dielectric anisotropy ($\gamma \ll 1$).

As seen in Figure 2 a stripe texture occurs if $k_2 < K$. In the opposite case ($k_2 > K$) only the Freedericksz transition takes place.

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