This article was downloaded by: [Tomsk State University of Control Systems and Radio]

On: 19 February 2013, At: 10:55

Publisher: Taylor & Francis

Informa Ltd Registered in England and Wales Registered Number: 1072954 Registered

office: Mortimer House, 37-41 Mortimer Street, London W1T 3JH, UK



Molecular Crystals and Liquid Crystals Incorporating Nonlinear Optics

Publication details, including instructions for authors and subscription information:

http://www.tandfonline.com/loi/gmcl17

Lifshitz Point in Phase Diagrams of Cholesteric Films in Electric Fields

Peter Schiller ^a

^a Sektion Chemie der Martin-Luther-Universität, 4010-, Halle, G.D.R.

Version of record first published: 22 Sep 2006.

To cite this article: Peter Schiller (1990): Lifshitz Point in Phase Diagrams of Cholesteric Films in Electric Fields, Molecular Crystals and Liquid Crystals Incorporating Nonlinear Optics, 192:1, 323-327

To link to this article: http://dx.doi.org/10.1080/00268949008035649

PLEASE SCROLL DOWN FOR ARTICLE

Full terms and conditions of use: http://www.tandfonline.com/page/terms-and-conditions

This article may be used for research, teaching, and private study purposes. Any substantial or systematic reproduction, redistribution, reselling, loan, sub-licensing, systematic supply, or distribution in any form to anyone is expressly forbidden.

The publisher does not give any warranty express or implied or make any representation that the contents will be complete or accurate or up to date. The accuracy of any instructions, formulae, and drug doses should be independently verified with primary sources. The publisher shall not be liable for any loss, actions, claims, proceedings, demand, or costs or damages whatsoever or howsoever caused arising directly or indirectly in connection with or arising out of the use of this material.

Mol. Cryst. Liq. Cryst. 1990, Vol. 192, pp. 323-327 Reprints available directly from the publisher Photocopying permitted by license only © 1990 Gordon and Breach Science Publishers S.A. Printed in the United States of America

LIFSHITZ POINT IN PHASE DIAGRAMS OF CHOLESTERIC FILMS IN ELECTRIC FIELDS

PETER SCHILLER

Sektion Chemie der Martin-Luther-Universität 4010-Halle, G. D. R.

Abstract A mathematical procedure for locating Lifshitz points in phase diagrams of field induced transitions in cholesteric films is presented.

INTRODUCTION

As well known, planar cholesteric films can exhibit either a Freedericksz transition or a transition to a stripe texture in electric fields. Periodic equilibrium structures are also found in non-twisted nematic layers with large elastic anisotropy.

Thus three phases have to be taken into consideration, namely the initial director configuration (I), a homogeneously distorted state (II) due to a Freedericksz transition and a modulated phase (III). Recently, Allender claimed³, that these three phases meet at a Lifshitz point of a suitably constructed phase diagram. In this communication a mathematical procedure for obtaining the position of Lifshitz point is presented.

Figure 1 illustrates the geometry of a planar cholesteric film. The director is fixed at the plate surfaces (x = 0 and x = d) and twisted by an azimuthal angle α . Below the threshold voltage of an instability the angle α enclosed by the director and the mid-plane of the film x = d/2 is zero.

The stripes are perpendicular to the wave vector ${\bf q}$ of the periodic distortions, ${\bf q}$ and the director at the lower plate enclose a definite angle ${\bf 22}$.

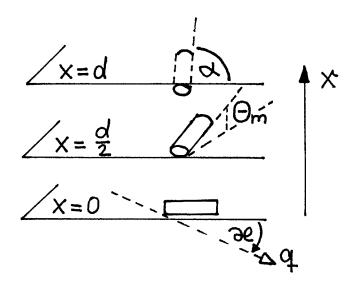


FIGURE 1 Geometry of a cholesteric layer
For convenience the abbreviations

$$k_2 = \frac{K_{22}}{K_{11}}$$
, $k_3 = \frac{K_{33}}{K_{11}}$, $\omega = \frac{\Delta}{3\tau}$

$$\beta = \frac{2 \text{ st d}}{P \text{ and}} \text{ and } \gamma = \frac{\mathcal{E}_{\parallel} - \mathcal{E}_{\perp}}{\mathcal{E}_{\perp}}$$

are introduced. $\rm K_{11},~K_{22}$ and $\rm K_{33}$ are elastic constants defined in the frame-work of the Oseen-Frank theory $^4.$

 ξ_{11} and ξ_{\perp} are dielectric constants measured parallel and perpendicular to the director, respectively.

PHASE DIAGRAM WITH LIFSHITZ POINT

Let us choose the applied voltage U and \mathbf{k}_2 as independent variables. If a Lifshitz point L exists, a phase diagram shown in Figure 2 can be constructed.

Three phases characterized by

(I)
$$\Theta_{\rm m}=0$$
 , (II) $\Theta_{\rm m}={\rm constant} \neq 0$

and a modulated one

(III)
$$\Theta_{\rm m} \sim \cos(qy)$$

meet at the point L. (The y-axis is parallel to q in Figure 1.)

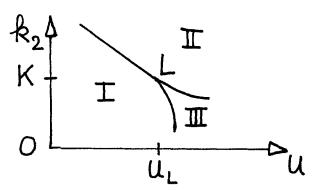


FIGURE 2 Phase diagram with Lifshitz point L

LOCATION OF THE LIFSHITZ POINT

The critical voltage at the Lifshitz point is

$$U_{L} = \sqrt{\frac{\pi^{2} \kappa_{11}}{\varepsilon_{11} - \varepsilon_{1}} \left[1 + \omega^{2} (k_{3} - 2K + 2K\beta) \right]}$$

and the critical value K of k_2 is obtained by a simple procedure.

Introducing an integral operator IN by the identity

In
$$(f(x)) = \frac{\pi - x}{\pi} \int_{0}^{x} f(\xi) d\xi - \frac{x}{\pi} \int_{0}^{\pi} (\pi - \xi) f(\xi) d\xi$$
we define

F(x) = In (f(x)) and G(x) = In (g(x))

wh**er**e

$$f(x) = \frac{1-K}{K} \sin \Omega \cos x - \frac{k_3-K+2K}{K} \omega \cos \Omega \sin x,$$

$$g(x) = \sqrt{-\cos \Omega} \sin x \quad \text{and}$$

$$\Omega = \omega x + 2e.$$

Now a function D (%) is obtained by integration

$$D(\mathcal{H}) = \frac{1}{\pi} \int_{0}^{\pi} dx \sin x \left\{ -(1-K) \sin \Omega \frac{dF}{dx} - (1-2K+k_3 + 2K\beta) \omega \cos \Omega F + (K \sin^2 \Omega + k_3 \cos^2 \Omega) \sin x - \left[1+\omega^2 (k_3-2K + 2K\beta) \right] \cos \Omega G \right\}.$$

Let $D(\mathcal{H})$ have its absolute minimum for $\mathcal{H} = \mathcal{H}_m$.

If \mathcal{H}_m obeys

$$\mathcal{H}_{m} = \begin{cases} 0 & \text{or } \frac{\pi}{2} & \text{if } \mathcal{O} = 0 \\ -\frac{\mathcal{O}}{2} + n\pi & \text{if } \mathcal{O} \neq 0 \end{cases}$$
(n is an integer number),

then a Lifshitz point exists and K is obtained by the equation

$$D(\partial \ell_m; K) = 0.$$

For example, if $\alpha = \beta = 0$, we get $\theta_m = \frac{\pi}{2}$ and $\alpha = 0$, we get $\theta_m = \frac{\pi}{2}$ and $\alpha = 0.303$ in accordance with a result of Lonberg and Meyer².

Secondly, let us regard a twisted cholesteric film with $\alpha=180^{\circ}$. In this case also results $\alpha=\pi/2$, and K satisfies the equation

$$4 K^2 + 12 Kk_3 + 8(1-K)^2 - 3 (k_3-1+2K\beta)^2$$

$$-2 \left(\frac{\pi^2+3}{3}\right) \left(k_3+1-2K+2K\beta\right)^2$$

+
$$(\frac{2\pi^2+15}{3})$$
 $(k_3+1-2K+2K\beta)$ $K\gamma = 0$.

It should be remarked, that a previously obtained formula 5 for K refers to a small dielectric anisotropy (>4<1).

As seen in Figure 2 a stripe texture occurs if $k_2 < K$. In the **opp**osite case $(k_2 > K)$ only the Freedericksz transition takes place.

REFERENCES

- 1. V. G. Chigrinov, V. V. Belyaev, S. V. Belyaev and M. F. Grebenkin, Sov. Phys., <u>JETP</u>, 50, 994 (1979)
- 2. F. Lonberg and R. B. Meyer, Phys. Rev. Lett., 55, 718 (1985)
- 3. D. W. Allender, Program and Abstracts of the Twelfth International Liquid Crystal Conference, Freiburg 1988, IN 16, p. 554
- 4. P. C. De Gennes, The Physics of Liquid Crystals, London 1974
- 5. P. Schiller, Liq. Cryst., to appear